

Two results on equations of nilpotent orbits.

by

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0. Introduction.

In this note I prove two results on the minimal generators of the defining ideals of closure of nilpotent conjugacy classes in the Lie algebra of $n \times n$ matrices. These results were motivated by questions of Pappas and Rapoport [PR] and they provide the answers to some of their conjectures.

In order to formulate the results let us recall the following notation.

Let K be a field and let E be a vector space of dimension n over K . We denote by $X = \text{Hom}_K(E, E)$ the set of endomorphisms of E . Let $\{e_1, \dots, e_n\}$ be a basis in E . We write the elements of X in the form

$$\phi(e_j) = \sum_{i=1}^n \phi_{i,j} e_i,$$

thus identifying X with a set of $n \times n$ matrices. We also identify the space $\text{Hom}_K(E, E)$ with $E^* \otimes E$. In this tensor product we identify ϕ with

$$\phi = \sum_{i,j=1}^n \phi_{i,j} e_j^* \otimes e_i.$$

The general linear group $GL(E)$ acts by conjugation on X . For a partition $\mu = (\mu_1, \dots, \mu_s)$ of n let X_μ be the closure of the set $O(\mu)$ of nilpotent matrices with Jordan blocks of sizes μ_1, \dots, μ_s . We denote by $A = K[X] = \text{Sym}(E \otimes E^*)$ the coordinate ring of X , and $J(\mu)$ denotes the defining ideal of X_μ . The coordinate functions $\Phi_{i,j}$ are the generators of A . They can be treated as independent indeterminates. We denote by

$$\Phi = (\Phi_{i,j})_{1 \leq i,j \leq n}$$

the generic $n \times n$ matrix.

Let us describe some important polynomials in A . Let I, J be two subsets of $[1, n]$ of cardinality r . Then $M(I, J)$ denotes the determinant of the $r \times r$ submatrix of Φ with rows from I and columns from J .

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We recall that the characteristic polynomial of ϕ can be written

$$P(\phi, x) = \sum_{i=0}^n T_i x^{n-i} ,$$

where $T_0 = 1$ and T_i is the sum of principal $i \times i$ minors of the generic matrix Φ ,

$$T_i = \sum_{I \subset [1, n], \text{card}(I)=i} M(I, I).$$

The polynomials T_1, \dots, T_n are the generators of the ring of $GL(E)$ -invariants in A .

We are interested in special nilpotent orbits. Let us fix a number $e < n$ and let $n = re + f$ be the division of n by e with the remainder f . To such n, e we associate the partition $\mu(n, e) = (e^r, f)$. It is clear that the set $X_{\mu(n, e)}$ can be defined set theoretically by the conditions $\phi^e = 0$. The subject of this note is the description of minimal generators of the ideal $J(\mu(n, e))$.

Theorem 1. *Let K be a field of characteristic zero. Then the ideal $J(\mu(n, e))$ is generated by the invariants T_1, \dots, T_{e-1} and by the entries of the matrix Φ^e . These polynomials form a minimal set of generators of $J(\mu(n, e))$.*

Theorem 2. *Let K be an arbitrary field and let $e = 2$. The ideal $J(\mu(n, 2))$ is generated by the invariants T_1, T_2, \dots, T_n and by the entries of the matrix Φ^2 .*

The proof of the first theorem is based on the geometric method used in [W]. The main result in [W] described a small but nonminimal set of generators of $J(\mu)$ for arbitrary μ . Applying the method to the smaller class of partitions $\mu(n, e)$ yields Theorem 1.

The proof of Theorem 2 relies on the paper [S] of Strickland. Using that result one can easily show that the ideal $J(n, 2)$ is generated by the polynomials described in Theorem 2 and by the minors of Φ of rank $\lfloor \frac{n}{2} \rfloor + 1$. It remains to show that the ideal generated by these minors is contained in the ideal generated by the polynomials described in Theorem 2. This is done by explicit calculation, using some information about two-rowed Schur functors in characteristic $p > 0$.

Throughout the paper we denote by $S_\mu E$ the Schur module corresponding to the highest weight μ for the group $GL(n)$. This means that $\mu = (\mu_1, \dots, \mu_n)$ where $\mu_i \in \mathbf{Z}$, $\mu_1 \geq \dots \geq \mu_n$.

1. The proof of Theorem 1.

Let E, K be as above. For a given number p , $1 \leq p \leq n$ we consider the subrepresentation of A which is the span of $p \times p$ minors of the generic matrix Φ . This vector space can be identified with $\bigwedge^p E \otimes \bigwedge^p E^*$. For arbitrary i , $0 \leq i \leq p$ we define the $GL(E)$ -equivariant map $\theta(i, p)$ to be the composition

$$\bigwedge^i E \otimes \bigwedge^i E^* \rightarrow \bigwedge^i E \otimes \bigwedge^i E^* \otimes K \xrightarrow{1 \otimes tr^{(p-i)}} \bigwedge^i E \otimes \bigwedge^i E^* \otimes \bigwedge^{p-i} E \otimes \bigwedge^{p-i} E^* \rightarrow \bigwedge^p E \otimes \bigwedge^p E^*$$

where the second map is tensoring with the $(p-i)$ 'st divided power of the trace element $tr = \sum_{i=1}^n e_i \otimes e_i^* \in E \otimes E^*$ and the third map is a product of two exterior multiplications.

We define $V_{i,p}$ to be the image of this map. When K is a field of characteristic zero it is well known that the space $\bigwedge^p E \otimes \bigwedge^p E^*$ has the following $GL(E)$ -equivariant decomposition into irreducible representations of $GL(E)$,

$$\bigwedge^p E \otimes \bigwedge^p E^* = \bigoplus_{j=0}^{\min(p, n-p)} S_{1^j, 0^{n-2j}, (-1)^j} E,$$

where $S_{1^j, 0^{n-2j}, (-1)^j} E$ denotes the irreducible representation (Schur functor) with the highest weight $(1^j, 0^{n-2j}, (-1)^j)$. We denote the summand $S_{1^j, 0^{n-2j}, (-1)^j} E$ occurring in $\bigwedge^p E \otimes \bigwedge^p E^*$ by $U_{j,p}$.

Notice that $U_{0,p} = V_{0,p}$ can be identified with the one dimensional subspace generated by the invariant T_p .

Lemma 1. *Let K be a field of characteristic zero. We have $V_{i,p} = \sum_{j=0}^{\min(i, n-p)} U_{j,p}$.*

Proof. This is an easy calculation after one observes that the highest weight vector in $\bigwedge^p E \otimes \bigwedge^p E^*$ of weight $(1^i, 0^{2n-i}, (-1)^i)$ is

$$w(i, p) = \sum_{J \subset [1, n], |J|=p-i} (e_1 \wedge \dots \wedge e_i \wedge e_J) \otimes (e_J^* \wedge e_{n+1-i}^* \wedge \dots \wedge e_n^*),$$

where for $J = \{j_1, \dots, j_t\}$ we write $e_J = e_{j_1} \wedge \dots \wedge e_{j_t}$, $e_J^* = e_{j_1}^* \wedge \dots \wedge e_{j_t}^*$. •

Remark.

- a) Notice that $U_{0,p} = V_{0,p}$ can be identified with the one dimensional subspace generated by the invariant T_p . The representation $V_{1,p}$ is the subspace generated by the entries of the p -th power Φ^p of the generic matrix.
- b) We have a commutative diagram

$$\begin{array}{ccc} \bigwedge^{i-1} E \otimes \bigwedge^{i-1} E^* & \xrightarrow{1 \wedge T_1} & \bigwedge^i E \otimes \bigwedge^i E^* \\ (p-i+1)\theta(i-1, p) \searrow & & \swarrow \theta(i, p) \end{array}$$

$$\bigwedge^p E \otimes \bigwedge^p E^*$$

which proves that $V_{i-1,p} \subset V_{i,p}$.

Before we start with the proof we need a couple of elementary lemmas.

Lemma 2. *Let $i \geq 1$. Then the vector space $V_{i,p+1}$ is contained in the ideal generated by $V_{i,p}$.*

Proof. Let us take a typical element of $V_{i,p+1}$. It can be written as

$$\sum_{j_1, \dots, j_{p+1-i}} M(u_1, \dots, u_i, j_1, \dots, j_{p+1-i}; v_1, \dots, v_i, j_1, \dots, j_{p+1-i}).$$

Here we are summing over all choices of j_1, \dots, j_{p+1-i} regardless of whether the summands are nonzero. Let us take the Laplace expansion of every term in the above sum, with respect to the row u_1 . This allows us to express our generator in the following form

$$\begin{aligned} & \sum_{j_1, \dots, j_{p+1-i}} \left(\sum_{k=1}^i (-1)^{k+1} \Phi_{u_1, v_k} M(u_2, \dots, u_i, j_1, \dots, j_{p+1-i}; v_1, \dots, \hat{v}_k, \dots, v_i, j_1, \dots, j_{p+1-i}) + \right. \\ & \left. + \sum_{l=1}^{p+1-i} (-1)^{i+l+1} \Phi_{u_1, j_l} M(u_2, \dots, u_i, j_1, \dots, j_{p+1-i}; v_1, \dots, v_i, j_1, \dots, \hat{j}_l, \dots, j_{p+1-i}) \right). \end{aligned}$$

Writing the first part of the sum as a linear combination of Φ_{u_1, v_k} we see that all coefficients are in $V_{i-1,p} \subset V_{i,p}$. Similarly, writing the second part of the sum as a linear combination of Φ_{u_1, j_l} we see that each coefficient is in $V_{i,p}$. •

We also recall the geometric method for calculating syzygies. We consider the Springer type desingularization Y_μ of X_μ . Let $\mu' = (\mu'_1, \dots, \mu'_t)$ be the partition conjugate to μ . Let $G/P_{\mu'}$ be the flag variety of partial flags $(R_1 \subset \dots \subset R_{t-1} \subset R_t = E)$ with $\dim R_i = \mu'_1 + \dots + \mu'_i$, and

$$Y_\mu = \{(\phi, R_1, \dots, R_t) \in X \times G/P_{\mu'} \mid \phi(R_i) \subset R_{i-1} \text{ for } i = 1, \dots, t\}.$$

We denote by \mathcal{R}_i the tautological vector bundle on $G/P_{\mu'}$ of dimension $\mu'_1 + \dots + \mu'_i$, corresponding to the subspace R_i . We denote by \mathcal{Q}_i the tautological factorbundle $(E \times G/P_{\mu'})/\mathcal{R}_i$.

The variety Y_μ is the total space of the cotangent bundle on $G/P_{\mu'}$. Let \mathcal{T}_μ denote the tangent bundle on $G/P_{\mu'}$. Then we have an exact sequence of vector bundles on $G/P_{\mu'}$

$$0 \rightarrow \mathcal{S}_\mu \rightarrow E \otimes E^* \times G/P_{\mu'} \rightarrow \mathcal{T}_\mu \rightarrow 0.$$

The bundle \mathcal{S}_μ is “the staircase bundle”. In terms of tautological bundles \mathcal{S}_μ can be described (as a subbundle of $(E \otimes E^*) \times G/P_{\mu'}$) as

$$\mathcal{S}_\mu = \mathcal{R}_1 \otimes \mathcal{Q}_0^* + \mathcal{R}_2 \otimes \mathcal{Q}_1^* + \dots + \mathcal{R}_t \otimes \mathcal{Q}_{t-1}^*$$

(the sum is not direct), with the convention $\mathcal{R}_t = E \times G/P_{\mu'}$, $\mathcal{Q}_0 = E^* \times G/P_{\mu'}$.

Recall that the basic theorem of the geometric method (comp. [W], [F-W]) implies the following

Theorem 3. *The varieties X_μ are normal, with rational singularities. Moreover, a minimal free resolution of the coordinate ring $K[X_\mu]$ as an A -module has the terms*

$$\dots \rightarrow F_i^\mu \rightarrow F_{i-1}^\mu \rightarrow \dots \rightarrow F_1^\mu \rightarrow F_0^\mu \rightarrow 0$$

where

$$F_i^\mu = \bigoplus_{j \geq 0} H^j(G/P_\mu, \bigwedge^{i+j} \mathcal{S}_\mu) \otimes_K A(-i-j).$$

where $A(-d)$ denotes the one dimensional homogeneous free A -module with a generator in degree d .

Let $\hat{\mu}$ be the partition $\hat{\mu} = (\max(\mu_1 - 1, 0), \max(\mu_2 - 1, 0), \dots, \max(\mu_t - 1, 0))$. Then $\hat{\mu}' = (\mu'_2, \dots, \mu'_t)$. We see that we have an exact sequence

$$0 \rightarrow E^* \otimes \mathcal{R}_1 \rightarrow \mathcal{S}_\mu \rightarrow \mathcal{S}_{\hat{\mu}}(\mathcal{Q}_1, \mathcal{Q}_1^*) \rightarrow 0 \quad (*)$$

where $\mathcal{S}_{\hat{\mu}}(\mathcal{Q}_1, \mathcal{Q}_1^*)$ is the bundle $\mathcal{S}_{\hat{\mu}}$ constructed in a relative situation with \mathcal{Q}_1 replacing E .

Using the exact sequence $(*)$ we can estimate the terms of the complex F_\bullet^μ inductively. We consider the minimal resolution $F^{\hat{\mu}}(\mathcal{Q}_1, \mathcal{Q}_1^*)_\bullet$ of the nilpotent orbit closure for the partition $\hat{\mu}$ in the relative situation (i.e. \mathcal{Q}_1 replaces E and we treat the resolution as the sequence of $\text{Sym}(\mathcal{Q}_1 \otimes \mathcal{Q}_1^*)$ -modules). For each term $S_\lambda \mathcal{Q}_1$ occurring in that resolution we can describe the push down $K_\lambda(E, E^*)$ of the complex $S_\lambda \mathcal{Q}_1 \otimes \bigwedge^\bullet(E^* \otimes \mathcal{R}_1)$. The terms of the complexes $K_\lambda(E, E^*)$ for all representations $S_\lambda \mathcal{Q}_1$ occurring in $F^{\hat{\mu}}(\mathcal{Q}_1, \mathcal{Q}_1^*)_\bullet$ give an upper bound on the terms from F_\bullet^μ .

The following facts are proved in [W], section 3.

Theorem 4. *Let $\mu, \hat{\mu}$ be as above. Let $S_\lambda \mathcal{Q}_1$ be the term in $F^{\hat{\mu}}(\mathcal{Q}_1, \mathcal{Q}_1^*)_i$. Then the terms coming from $K_\lambda(E, E^*)$ appear in homological degrees $\geq i$, i.e. they can give the contribution only to the terms F_j^μ with $j \geq i$.*

In particular, as stated in Corollary (3.15) of [W], the term F_0^μ is trivial, consisting of a copy of A in homogeneous degree 0. The terms in F_1^μ are, by Theorem (3.12) of [W], of two types:

1. The term $F^{\hat{\mu}}(\mathcal{Q}_1, \mathcal{Q}_1^*)_0$ is trivial, so the corresponding complex $K_{(0)}$ is just a pushdown of $\bigwedge^\bullet(E^* \otimes \mathcal{R}_1)$. Its first term gives a possible contribution to F_1^μ which is the vanishing of $n - \mu_1 + 1$ minors of the matrix Φ ,
2. Each term $S_\lambda \mathcal{Q}_1$ from $F^{\hat{\mu}}(\mathcal{Q}_1, \mathcal{Q}_1^*)_1$ gives a possible contribution to F_1^μ which is the zero'th term of the corresponding complex K_λ .

Now let us use the above inductive procedure for the family of partitions $\mu(n, e) = (e^r, f)$. The partition $\hat{\mu}(n, e) = ((e-1)^r, \max(f-1, 0))$ is just $\mu(n-r-1, e-1)$ for $f > 0$

or $\mu(n-r, e-1)$ for $f=0$. These partitions are in the same family, so we can assume by induction that Theorem 1 is true for $\hat{\mu}$.

The inductive assumption means that the first term of $F_1^{\hat{\mu}(n,e)}$ consists of the trivial representations in homogeneous degrees $1, \dots, e-1$ and the representation $S_{1,0,\dots,0,-1}\mathcal{Q}_1$ in homogeneous degree $e-1$. This means that the terms in F_1^μ have trivial representations in homogeneous degrees $1, \dots, e-1$, the representation $E^* \otimes E$ in degree e (comp. Lemma (3.11) in [W]) and the terms coming from $n - \mu(n, e)_1 + 1 = n - r$ size minors of the matrix Φ . The generators in degrees $\leq e$ have to consist of the invariants T_1, \dots, T_{e-1} and the vanishing of entries of Φ^e , because these terms have to occur in F_1^μ and they match the terms we described. Thus the induction implies that the minimal generators of $J(\mu(n, e))$ are those listed in Theorem 1 plus possibly some linear combinations of $(n-r) \times (n-r)$ minors of Φ .

However we also recall what the Theorem (4.6) from [W] says about the ideal $J(n, r)$.

Theorem 5. *The ideal $J(\mu(n, e))$ is generated (nonminimally) by the invariants $U_{0,p}$ ($1 \leq p \leq n$) and by the representations $U_{i,ie-i+1}$ (for $1 \leq i \leq r$).*

The only possible generator in degree $n-r$ on the above list is the invariant $U_{0,n-r}$. However this cannot be a minimal generator because $U_{0,n-r}$ is contained in $V_{1,n-r}$ which is in the ideal generated by $V_{1,e}$ according to Lemma 2. This concludes the proof of Theorem 1.

2. The proof of Theorem 2.

In this section we work over a field K of arbitrary characteristic. We are interested in the generators of the ideal $J(\mu(n, 2))$. Let us recall that Strickland in [S] exhibited a standard basis in the coordinate ring $A/J(\mu(n, 2))$ of the orbit closure $X_{\mu(n, 2)}$. Denote by $M(a_1, \dots, a_r; b_1, \dots, b_r)$ the $r \times r$ minor of Φ corresponding to rows a_1, \dots, a_r and columns b_1, \dots, b_r . One notices easily that the only relations used in [S] to prove that the standard monomials span the ring $A/J(\mu(n, 2))$ are the relations

$$\sum_{1 \leq i_1 < \dots < i_r \leq n} M(a_1, \dots, a_{p-r}, i_1, \dots, i_r; b_1, \dots, b_{p-r}, i_1, \dots, i_r) \quad Rel(r, p)$$

for $1 \leq p \leq n, 1 \leq r \leq p$ and $(\lfloor \frac{n}{2} \rfloor + 1) \times (\lfloor \frac{n}{2} \rfloor + 1)$ minors of Φ . Therefore Theorem 2 is a consequence of the following lemmas.

Lemma 3. *The polynomials of type $Rel(r, p)$ for $1 \leq p \leq n, 1 \leq r \leq p$ are in the ideal generated by the invariants T_1, T_2, \dots, T_n and by the entries of Φ^2 .*

Proof. Let us denote by J the ideal generated by the invariants T_1, \dots, T_n and by the entries of Φ^2 . We use induction on p . For $p=1, 2$ we see that the equations of type $Rel(r, p)$ describe exactly T_1 and the entries of Φ^2 . Let us assume that the statement is

true for p . Let us first assume that $r > 1$. If $p - r = 0$ we are dealing with the invariant T_p , so it is in J . If $p - r > 0$ we see that after taking the Laplace expansion of $Rel(r, p+1)$ with respect to the row a_1 we get a linear combination of terms of type $Rel(r-1, p)$ so we are done by induction. For $r = 1$ we note that

$$\begin{aligned} & \sum_{1 \leq i \leq n} M(a_1, \dots, a_p, i; b_1, \dots, b_p, i) = \\ &= \sum_{1 \leq i \leq n} \sum_{1 \leq u \leq p} (-1)^{u+1} M(a_1; b_u) M(a_2, \dots, a_p, i; b_1, \dots, \hat{b}_u, \dots, b_p, i) + \\ & \quad + (-1)^p \sum_{1 \leq i \leq n} M(a_1; i) M(a_2, \dots, a_p, i; b_1, \dots, b_p). \end{aligned}$$

The first summand on the right hand side is in J by induction. It remains to deal with the second summand which we will denote by F . We take the Laplace expansion with the respect to the row i in each term in that summand. We obtain

$$F = \sum_{1 \leq i \leq n} \sum_{1 \leq v \leq p} (-1)^{v+1} M(a_1; i) M(i; b_v) M(a_2, \dots, a_p; b_1, \dots, \hat{b}_v, \dots, b_p).$$

However

$$\sum_{1 \leq i \leq n} M(a_1; i) M(i; b_v) = \sum_{1 \leq i \leq n} M(i; i) M(a_1; b_v) + \sum_{1 \leq i \leq n} M(a_1, i; b_v, i) .$$

Since the second summand above is in J , we see that in A/J we have

$$\begin{aligned} F &= \sum_{1 \leq i \leq n} M(i, i) \sum_{1 \leq v \leq p} (-1)^{v+1} M(a_1, b_v) M(a_2, \dots, a_p; b_1, \dots, \hat{b}_v, \dots, b_p) = \\ &= T_1 M(a_1, \dots, a_p; b_1, \dots, b_p) \end{aligned}$$

so $F \in J$ and we are done.

Lemma 4. *The $(\lfloor \frac{n}{2} \rfloor + 1) \times (\lfloor \frac{n}{2} \rfloor + 1)$ minors of Φ are contained in the ideal spanned by the invariants T_1, T_2, \dots, T_n and by the entries of the matrix Φ^2 .*

Proof. We employ the following notation. We write $m = \lfloor \frac{n}{2} \rfloor + 1$. We identify the set of $m \times m$ minors with $\bigwedge^m E \otimes \bigwedge^m E^* = \bigwedge^m E \otimes \bigwedge^{n-m} E$. This is only an $SL(E)$ -isomorphism but it is sufficient for our purposes. The relation $Rel(r, m)$ translates under this identification to the image of the following map.

$$\psi(r, m) : \bigwedge^{m-r} E \otimes \bigwedge^{n-m+r} E \xrightarrow{1 \otimes \Delta} \bigwedge^{m-r} E \otimes \bigwedge^r E \otimes \bigwedge^{n-m} E \xrightarrow{m \otimes 1} \bigwedge^m E \otimes \bigwedge^{n-m} E$$

Therefore we need to prove

Lemma 5. *The vector space $\bigwedge^m E \otimes \bigwedge^{n-m} E$ is spanned by the images of the maps $\psi(r, m)$ for $1 \leq r \leq m$.*

Proof. We use induction on n . We prove the spanning weight by weight. Notice that the only weights in $\bigwedge^m E \otimes \bigwedge^{n-m} E$ are (up to permutation of the basis) the weights $(2^j, 1^{n-2j}, 0^j)$. Notice that the problem of spanning in the weight $(2^j, 1^{n-2j}, 0^j)$ for a given n is equivalent to the problem of spanning in the weight (1^{n-2j}) for the dimension $n - 2j$. So by induction on n we can assume that all vectors of weights $\neq (1^n)$ are in the subspace generated by the images of $\psi(r, m)$. But the subrepresentation of $\bigwedge^m E \otimes \bigwedge^{n-m} E$ generated by all vectors of weights $\neq (1^n)$ is isomorphic to the kernel of the exterior multiplication

$$\bigwedge^m E \otimes \bigwedge^{n-m} E \rightarrow \bigwedge^n E.$$

Therefore it is enough to show that the images of $\psi(r, m)$ cannot be all contained in this kernel. However one sees easily that the map $\psi(r, m)$ composed with the exterior multiplication is the exterior multiplication

$$\bigwedge^{m-r} E \otimes \bigwedge^{n-m+r} E \rightarrow \bigwedge^n E$$

multiplied by the binomial coefficient $\binom{n-m+r}{r}$. So Theorem 2 follows from the last elementary lemma.

Lemma 6. *Let n be an integer, $m = \lfloor \frac{n}{2} \rfloor + 1$. Then*

$$GCD_{1 \leq r \leq m} \binom{n-m+r}{r} = 1.$$

Proof. Let $n = 2t$ be even. Then $m = t + 1$. We are interested in

$$GCD_{1 \leq r \leq t+1} \binom{t-1+r}{r}.$$

Using the relations $\binom{k}{r} = \binom{k-1}{r} + \binom{k-1}{r-1}$ we get

$$GCD_{2 \leq r \leq t+1} \binom{t-1+r}{r} = GCD_{2 \leq r \leq t+1} \binom{t+1}{r}$$

which obviously equals to 1.

If $n = 2t + 1$ is odd, then $m = t + 1$ and we are interested in

$$GCD_{1 \leq r \leq t+1} \binom{t+r}{r}.$$

Again using the same method we find that

$$GCD_{1 \leq r \leq t+1} \binom{t+r}{r} = GCD_{1 \leq r \leq t+1} \binom{t+1}{r} = 1.$$

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